

# NAG Toolbox for MATLAB

## f07fg

### 1 Purpose

f07fg estimates the condition number of a real symmetric positive-definite matrix  $A$ , where  $A$  has been factorized by f07fd.

### 2 Syntax

```
[rcond, info] = f07fg(uplo, a, anorm, 'n', n)
```

### 3 Description

f07fg estimates the condition number (in the 1-norm) of a real symmetric positive-definite matrix  $A$ :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since  $A$  is symmetric,  $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$ .

Because  $\kappa_1(A)$  is infinite if  $A$  is singular, the function actually returns an estimate of the **reciprocal** of  $\kappa_1(A)$ .

### 4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

### 5 Parameters

#### 5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates how  $A$  has been factorized.

**uplo** = 'U'

$A = U^T U$ , where  $U$  is upper triangular.

**uplo** = 'L'

$A = L L^T$ , where  $L$  is lower triangular.

*Constraint:* **uplo** = 'U' or 'L'.

2: **a(lda,\*)** – double array

The first dimension of the array **a** must be at least  $\max(1, \mathbf{n})$

The second dimension of the array must be at least  $\max(1, \mathbf{n})$

The Cholesky factor of  $A$ , as returned by f07fd.

3: **anorm** – double scalar

The 1-norm of the **original** matrix  $A$ , which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07fd or else from a copy of the original matrix  $A$ .

*Constraint:* **anorm**  $\geq 0.0$ .

## 5.2 Optional Input Parameters

1: **n** – int32 scalar

*Default:* The second dimension of the array **a**.

*n*, the order of the matrix *A*.

*Constraint:*  $n \geq 0$ .

## 5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, iwork

## 5.4 Output Parameters

1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of *A*. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, *A* is singular to working precision.

2: **info** – int32 scalar

**info** = 0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** =  $-i$

If **info** =  $-i$ , parameter *i* had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **anorm**, 6: **rcond**, 7: **work**, 8: **iwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

## 7 Accuracy

The computed estimate **rcond** is never less than the true value  $\rho$ , and in practice is nearly always less than  $10\rho$ , although examples can be constructed where **rcond** is much larger.

## 8 Further Comments

A call to f07fg involves solving a number of systems of linear equations of the form  $Ax = b$ ; the number is usually 4 or 5 and never more than 11. Each solution involves approximately  $2n^2$  floating-point operations but takes considerably longer than a call to f07fe with one right-hand side, because extra care is taken to avoid overflow when *A* is approximately singular.

The complex analogue of this function is f07fu.

## 9 Example

```
uplo = 'L';
a = [4.16, -3.12, 0.56, -0.1;
     -3.12, 5.03, -0.83, 1.18;
     0.56, -0.83, 0.76, 0.34;
     -0.1, 1.18, 0.34, 1.18];
```

```
anorm = norm(a, 1);  
[a, info] = f07fd(uplo, a);  
[rcond, info] = f07fg(uplo, a, anorm)
```

```
rcond =  
      0.0103  
info =  
      0
```

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